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ABSTRACT

This paper presents an inquiry into teaching and learning practices in a Year 11 mathematics class at a private college for girls in Western Australia, studying a topic on vectors. The focus is on learning through problem-solving, where students used graphics calculators as a matter of routine. Constructivist and sociocultural theories were referents for the inquiry, and purposive selection of data led to consideration of a range of theoretical issues. The following five key classroom actions are considered from both constructivist and sociocultural perspectives: (1) answering questions in whole-class work; (2) drawing diagrams; (3) answering friends' questions; (4) trying things out on a graphics calculator and explaining to the class; and (5) listening to the teacher in whole-class work. The sometimes inconsistent assumptions of the two perspectives allowed complementary insights into teaching and learning, thus enriching the analysis. The critical and inquiring stance raises questions for mathematics teachers looking to achieve a sensitive and inclusive learning environment characterized by reflective mathematical thinking and rich mathematical conversations. Appendices include a copy of a student questionnaire and a graphics calculator activity worksheet. Contains 44 references. (MES)

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Enactment of Learning in the Presence of Graphics Calculators

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This paper presents an inquiry into teaching and learning practices in a Year 11 class studying a topic on vectors. The focus is on learning through problem-solving, where students used graphics calculators as a matter of routine. Constructivist and sociocultural theories were referents for the inquiry, and purposive selection of data led to consideration of a range of theoretical issues. The sometimes inconsistent assumptions of the two perspectives allowed complementary insights into teaching and learning, thus enriching the analysis. Our critical and inquiring stance raises questions for mathematics teachers looking to achieve a sensitive and inclusive learning environment characterized by reflective mathematical thinking and rich mathematical conversations.

Introduction

This paper is part of a larger study set in a senior high school (Year 11) mathematics class where students employed graphics calculators in their study of a topic on vectors. Here, we consider student-student and teacher-student dialogue, students' use of graphics calculators and other actions associated with learning through problem-solving. We use evidence from a lesson that commenced with the teacher working a problem along with the class and then later changed to small-group problem-solving. Viewing learning as a dynamic process, we move back and forth in time to link the action in the lesson with learning outcomes from other days. Problem-solving practices are highly relevant to teaching reform, with different approaches to problem-solving being one explanation (U.S. Department of Education, 1999) for the superior performance in mathematics by Japanese students to that of American and German students in the Third International Mathematics and Science Study (TIMSS).

The key epistemic referents for our inquiry were constructivist theory, where the primary emphasis is on *personal cognition*, and sociocultural theory, where learning is understood as a process of *enculturation* or *coming to know* in a community of practice. Our purpose in writing the paper was to explore implications for more effective senior high school mathematics teaching by interpreting critically, in terms of this dual theoretical perspective, a variety of learning practices that were enacted in a Year 11 class. Values we brought to the inquiry and which can be seen in our interpretive analysis are that teaching should ideally be inclusive of all students and students' ideas and creativity should be respected and encouraged.

Our analysis is centred on selected teacher and student conversations. We do not claim the excerpts of conversation represent regular practices among the class, but considering *the particular* (conversations and associated actions) in terms of *the general* (theory) allowed us to explore instructional/pedagogical questions which we believe have wide relevance to teaching and learning mathematics. Students' interpersonal interactions, their interactions with graphics calculators, and others of their mathematising actions are discussed in terms of *reflective* knowing, consistent with constructivist theory, and in terms of *reflexive* (loosely automatic) enculturation, associated with sociocultural theory.

An important feature of the paper for mathematics education researchers is the dual theoretical approach. Rather than considering cognitive and sociocultural views of learning as competing, one to the exclusion of the other, which is the thrust of recent debate in the literature (Anderson, Reder & Simon, 1997; Greeno, 1997), our analysis supports Sfard's (1998) recommendations that it is desirable and even necessary to consider the *paradigm of complementarity*, thus allowing an enriched critique of empirical data to inform the reform of teaching. Our inquiry uniquely takes this dual perspective into a senior high school mathematics classroom where students are using graphics calculators for the study of vectors.

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Each student owned a graphics calculator so use of the technology was embedded in the curriculum, rather than being a temporary innovation.

Dual Theoretical Viewpoint

Our inquiry was informed by theories of *constructivism* (Cobb, Gravemeijer, Yackel, McClain & Whitenack, 1997; Cobb, Boufi, McClain & Whitenack, 1997; Cobb & Bowers, 1999; Noddings, 1990; Voigt, 1994; von Glasersfeld, 1987) and by *sociocultural perspectives*, including social constructionism (Gergen, 1995) and situated cognition theory (Lave & Wenger, 1991).

In an extensive review of types of constructivism, Ernest (1995) states that in the *radical constructivist* view "the cognizing subject is a creature with sensory inputs, furnishing data that are interpreted (or rather constructed) through the lenses of its cognitive structures" (p. 474). The emphasis is on personal cognising--building on existing knowledge, where a metaphor for the mind is an "evolving, adapting organism" (p. 482); and thus mind is taken as embodied within the individual, cognizing being. *Reflection*, or Piaget's reflective abstraction, is an essential element of the development of understanding (Noddings, 1990). Learning might entail co-construction with others (Steffe, 1995) but ultimately knowledge is personal and self-constructed, where the criterion for viability is that knowledge is feasible in a given setting--it must 'work' in providing a basis for an individual's appropriate action (Davis, 1996). Thus, our constructivist perspective on teaching and learning mathematics focused on the provision of opportunities for students to engage in *reflective mathematical thinking* as they considered the viability of their extant understandings, strove to resolve creatively new cognitive (and emotional) perturbations, and tested the viability of their tentative solution strategies.

By contrast, sociocultural views prioritise the social above the individual: "Mind is regarded as the introjected social dimension. To put it another way, evidence of the mental is to be found in social performance" (Ernest, 1995, p. 481). Constructionist metaphors construe mind as distributed in dialogical space and learners as actors in a drama. Meaning is achieved through social interdependence so that a statement "is not author determined, but the product of intertextuality" (Gergen, 1995, p. 22), with all parties to a conversation influencing the meaning. What is known is context-dependant and learning mathematics constitutes reflexive enculturation into mathematising practices, such as might occur in an apprenticeship (Lave & Wenger, 1991). Our sociocultural perspective focused on the provision of opportunities for students to co-construct valid mathematical meaning through participation (in varying social roles) in *rich mathematical conversations* with the teacher, graphics calculators and fellow students.

Constructivist and sociocultural theories have their respective adherents, many of whom wish to argue for the superiority of their particular worldview (Anderson, Reder & Simon, 1997; Greeno, 1997), resulting in a stand-off not unlike that between those who extol the virtues of Macs or PCs. However, we prefer a transcendent and synergistic standpoint akin to Sfard's (1998) *paradigm of complementarity* or Cobb's (1994) *theoretical pragmatism*, which values the educational benefits to be had by coordinating two theoretical referents in the one inquiry. Such a combination can provide a unique view of the tension created by the dilemma of teaching for both individual understanding and enculturation. In this study, we used both constructivist and sociocultural perspectives in a complementary way to generate insights into the individual and social processes of mathematical problem-solving. We do remain aware, however, that these are not the only perspectives available for making sense of what goes on in the school mathematics classroom or, indeed, for arguing what should (or should not) take

place; for that purpose we would need to load in other (perhaps critical) theories of teaching and learning and schooling.

Research Methodology

Our inquiry was shaped by a *constructivist interpretative* methodology (Noddings, 1990), with each stage of the inquiry iteratively informing the next. Initially, data were generated through the personal observations of the first author. Subsequently classroom conversations--associated particularly with mathematical problem solving--were selected for analysis, and a process of critical reinterpretation followed. At this stage, the principle (and recursive) means of inquiry included writing (Richardson, 1998) and discussion between the two authors.

The site of the research was a Year 11 mathematics class of 18 students in a private college for girls in Western Australia. The first author adopted the role of participant-observer (Atkinson & Hammersley, 1994), to observe whole-class discussion and serve as an assistant teacher during small-group work (and is indicated in transcript excerpts as 'Mrs F'). The main source of data for this paper was audio-recorded classroom conversation. Three small audio-recorders were placed on students' desks for the lesson and captured, we assume, most of what was said by seven students in small-group work, as well as the voices of the teacher (Mr C) and students during whole-class discussion. Being recorded was not a novelty: the conversations of five of the students had been recorded for at least 10 of the 14 previous observed lessons, and conversations of the other two students of the seven had been recorded for four other lessons. The audio-recordings were transcribed.

Other data formed a backdrop to the analysis presented in this paper. They were generated during a month of fieldwork and included the first author's field notes and journal entries, a video-recording of each lesson from a corner of the classroom, photocopies of students' written work, students' written feedback on a practical activity, and students' responses to a questionnaire (see Appendix 1). The questionnaire was administered in class four days before the lesson that is discussed here took place. The questionnaire items were based on an analysis of recorded classroom conversations from early in the observation period and were informed by the Student Negotiation scale on the Constructivist Learning Environment Survey, or CLES (Taylor, Fraser & Fisher, 1997). The items guided our identification of data from the audio-transcripts for this paper. Then, after preliminary analysis, we narrowed our choice, purposively selecting (Cohen & Manion, 1994) conversation excerpts that portrayed diversity in students' modes of problem-solving, particularly approaches which involved the use of graphics calculators.

To ensure quality in the research, we adhered to some of Guba and Lincoln's (1989) quality criteria for interpretative research. For *confirmability*, data are trackable to their sources. *Credibility* was pursued through: persistent observation during the lesson; clarifying with the teacher and the students, near the time of the lesson, any queries that arose after transcribing the audio-tapes; making explicit our intention in the selection of data; stating the critical and value stances taken jointly by us; engaging in critical dialogue about each version of the paper, which led to reinterpretation and taking turns in revising the paper through writing; and two teaching colleagues reading the paper to check that our interpretation of students' conversations was reasonable. While the paper has been jointly written, the use of first person 'I' and 'my' refers to the first author, especially in her role as participant-observer.

We did not, however, conduct *member checks* (Guba & Lincoln, 1989) to optimize the credibility of our inferences about the classroom actions of the teacher and students. That is, we did not ask the teacher and students to comment on the validity of our portrayal of their actions. There was an important reason for not doing so. Because our social contract with the

teacher involved only participant-observation research, and not formal intervention for the purpose of achieving teaching reform (as in stimulating the teacher's critical reflective thinking about the efficacy of his teaching), we felt that it was unethical to present him and his students with assertions suggesting that his actions may be compromising student learning. We preferred to maintain a harmonious relationship with the teacher. Nevertheless, in her role as co-teacher, the first author did on several occasions, in accordance with the criterion of *educative authenticity* (Guba & Lincoln, 1989), discuss informally with the teacher his apparent (and unwitting) practice of acting differentially towards students, that is, of not being inclusive in the provision to students of equal opportunities to engage with him in mathematical conversation.

Although our inferences about the teacher's and students' classroom actions were driven largely by our theoretical predisposition, we argue that these tentative inferences are, to a satisfactory degree, warranted empirically. In addition to claiming (limited) credibility, we argue that, because of the extensive involvement in the inquiry of the first author as a co-teacher who interacted closely with students in a tutorial role, our inferences about students' learning, especially their patterns of mathematical thinking, are *practically reasonable* (Fenstermacher, 1994).

Thus, in writing this paper, we have taken two steps to ensure that the epistemic status of our (theoretically driven) inferences is consistent with the quality of the empirical evidence. Importantly, we have avoided imputing intention (via interpretive inference) to the teacher's actions beyond that which would be self-evident to most mathematics education observers. And we have presented our inferences not so much as propositional assertions, which tend to emphasise accuracy of portrayal of participants' meaning perspectives (Erickson, 1986), but as critical questions designed to provoke the reader's *pedagogical thoughtfulness* (van Manen, 1990) about key issues of teaching and learning mathematics.

Working a Problem Along with the Class

In this section, we focus on five key classroom actions demonstrated by students during the first 11 minutes of a lesson whilst being led by the teacher in 'working a problem along with the class': (1) answering questions in whole-class work, (2) drawing diagrams, (3) answering friends' questions, (4) trying things out on a graphics calculator and explaining to the class, and (5) listening to the teacher in whole-class work. Each of these classroom actions is considered from both constructivist and sociocultural perspectives.

The lesson took place on March 26th, which is a point of reference for data from other days. It was structured around students working through textbook (Sadler, 1993) problems on two-dimensional vectors. The class had started the vector topic nearly one month previously, at the beginning of the research observation period. Conversation segments associated with working one particular problem are presented and accompanied by interpretative commentary that places the action in the context of previous lessons, then our interpretation of the segments of conversation in terms of theory follows.

The lesson started with teacher-initiated whole-class discussion on a question (see below) involving *position vectors*, that is, vectors that have a fixed location in space. On a graph, position vectors are drawn to start at the origin. The usual notion of vectors is that they are 'free', so can be drawn to start anywhere. A diagram illustrating the position vectors of the question is provided in Figure 1, but was not available to students as part of the question.

Points A, B, C and D have position vectors **i** + 2**j**, 4**i** - 2**j**, -**i** + 11**j** and 6**i** - 13**j** respectively. Find (a) $\vec{AB}$, (b) $\vec{BC}$, (c) $\vec{CD}$, (d) a vector in the same direction as $\vec{AB}$ but equal in magnitude to $\vec{CD}$. (Sadler, 1993, p. 76)

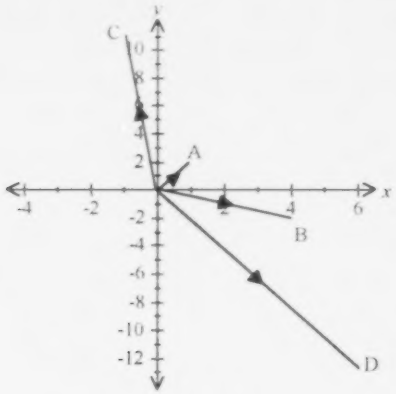

Figure 1. Diagram illustrating position vectors for the problem students were working on.

Answering Questions in Whole-class Work

The teacher read aloud the start of the question then started discussion on it.

Mr C: Now just to clear one thing up, each one of these can be written as a position vector. What does that mean? What does position vector mean?

Gemma: It means they can be plotted as points on a graph, with *x* and *y* co-ordinates. Like they are points on the graph.

Mr C: Okay. Everyone agree with that from the other day? Each one of those can be written as a position vector. So you should be able to handle a, b, and c.

On hearing the opening question, it seemed to me (first author) to relate to a challenge by a student to the teacher's definition of position vectors on introducing the concept two days previously. The teacher had plotted the point (3, -7) on the board to represent the position vector 3**i** -7**j**. A student asked "So a vector is a dot?" (audio-transcript, March 24th). The teacher's reply was "No. The position vector locates position, so what should I actually draw?" The student responded "An arrow" (see Figure 1), and the teacher agreed.

Constructivist view. Students' prior knowledge should be the starting point of instruction (Noddings, 1990) and the teacher's opening question ostensibly met this recommendation and encouraged students to individually 'look back' or reflect on their previous understanding of position vectors. However, if review questions are deemed important enough to ask, might not input by several students be warranted to ascertain any divergence in understanding? And, in view of the previous challenge, was the teacher justified in accepting Gemma's less than comprehensive response to his explicit question and in moving on? Her unelaborated explanation could have constituted for the class tacit acceptance of the convention that the graphical detail of arrows is not always included when drawing position vectors--sometimes only the endpoints of position vectors are plotted to save clutter on a diagram. But the important distinction that a point is not actually a vector was not then made by the teacher who seemed to overlook the potential problem of students having 'misunderstood' (Voigt, 1994)--a constructivist view is that understanding is individual and subjective. Mathematical relationships, like all concepts, are seen as personally construed, sometimes in working with others but the link "between discourse and psychological processes like reflective abstraction is indirect" (Cobb, Boufi, et. al., 1997, p. 264).

Sociocultural perspective. In the view that knowing resides in practices and participation (Lave & Wenger, 1991), Gemma by her assertiveness in answering the teacher's question, was actively engaged in the knowing (how and what to answer) and learning of the lesson. Interaction between Gemma and the teacher could be taken to have established meaning between them: "issues such as 'what we know' and 'who we are' are dialogical" (Davis, p. 187, 1996) and "the link between social processes and individual development is a direct one" (Cobb, Gravemeijer, et. al., 1997, p. 152). But did understanding extend to students who were (passively) listening?

As to the nature of understanding, the sociocultural view is that it is subjective, except that *social constructionists* "are alone in their rejection of the subjectivity of linguistic meaning" (Steffe, 1995, p. 491). However, from a social constructionist viewpoint the relationship of language to its referents involves negotiation (Gergen, 1995). So, whether we assume that understanding is subjective or objective, we might ask: did the teacher's omission to negotiate finer detail hide disagreement amongst the class as to the concept of position vectors? No-one answered when the teacher asked (rhetorically): "Everyone agree with that from the other day?"

Drawing Diagrams

Students had started to work on the problem when the teacher said to the class:

Mr C: The question arises here, we could spend ages doing diagrams, but do we need a diagram?
Jenny: No, but it helps to see what you are doing.
Mr C: But remember we want a bit of efficiency, a bit of speed.
Katie: I suppose so [to herself].

Jenny and Katie always sat together and their workbooks and recorded conversations showed that they regularly relied on diagrams to solve vector problems. For example, "I have to redraw the diagrams [from the text]. They are too small and I can't see what to do" (Jenny, March 12th) and "We are drawing a picture, because we don't know what else to do" (Katie, March 18th). They typically worked co-operatively and one problem solving method they used was "Scribbling [on a diagram]. Trying to get angles and things" (Jenny, March 26th).

Constructivist view. Meaningful learning is associated with conceptual understanding that is rich in relationships (Hiebert & Carpenter, 1992). Vector relationships can be expressed algebraically or geometrically, and might link these two forms. Having previously introduced the concept of position vectors with a visual (geometric) approach, the teacher was now encouraging students to omit the diagram and use only the algebra algorithm for finding the difference between position vectors. Algorithmic approaches to processing or calculation can have the advantage of being efficient and allowing students to concentrate on other aspects of problems (Eylon & Linn, 1988). However, should the response of Jenny, who was a top performing student, have been taken by the teacher as a signal that students still needed the support of diagramming so as to understand the order in which to subtract vectors algebraically. That otherwise students might rote memorise the subtraction property, for example that $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$, without being able to link the meaning of algebraic subtraction back to vector diagrams for the purposes of problem solving. Davis (1996) recommends careful hermeneutic listening to students: "a teaching that is attentive, creative, improvisional, hermeneutic--a playing by ear" (p. 225).

Sociocultural perspective. Lave and Wenger (1991) suggest metaphors of teacher as master craftsman and students as apprentices and, in a similar vein, Schön (1987) describes a teacher as coach and students as the team:

> The student cannot be *taught* what he needs to know, but he can be *coached*: 'He has to *see* on his own behalf and in his own way the relations between means and methods employed and results achieved.

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> Nobody else can see for him, and he can't see just by being 'told', although the right kind of telling may guide his seeing and thus help him see what he needs to see. (Dewey, cited in Schön, p. 17)

Jenny's and Katie's responses to the teacher as expert or coach can be seen to reflect and defend their tendencies to use diagrams in their problem-solving. The teacher did not outrule their preferences but they did respond, at least in the short term, to his guidance or coaching. perusal of their workbooks showed that both students drew a diagram for the question being discussed, then Jenny omitted them for the next seven questions and Katie for the next five. However, looking beyond this surface feature of the students' endeavour, we ask was the task on which they were working authentic (Roth & Tobin, in press)? Was it typical of problem-solving in the mathematical social world outside school? The textbook questions, including the one given above, were decontextualised and involved repetitive subtraction with only slight variations. Such drill and practice might be justified on the basis of skill improvement but generally is not associated with real-life problem-solving outside the classroom, or with deeply meaningful learning.

Answering Friends' Questions

Students were working on the first question, individually or with their neighbours, according to their own choice, when Amanda leant over and looked at Gemma's work:

Amanda:	What have you done now?
Gemma:	Oh, [pause] I have read the question wrong. I did AB [part (a) of the question]. Oh, what did I do?
Amanda:	AB?
Gemma:	No, I didn't do AB. What did I do? [pause] Take 4. Yes, that should be take 4. [Gemma's observation suggests she had written $3\mathbf{i} + 0\mathbf{j}$ instead of $3\mathbf{i} - 4\mathbf{j}$, making th mistake of saying that $4\mathbf{i}-2\mathbf{j} - (\mathbf{i}+2\mathbf{j}) = 3\mathbf{i}+0\mathbf{j}$]

During the first few lessons that I observed the class, Amanda sat next to Katie who had Jenny on her other side. When I asked him, the teacher said this had been their seating arrangement for the month since the beginning of the year. The audio-transcripts show that Amanda rarely talked to the other two students and when I was checking if they had solved a problem in similar ways, Amanda commented to me "I don't work with them" (audio-transcript, March 17th). After this, Amanda tried sitting with various other students and eventually settled on sitting next to Gemma, who already had a friend to work with but allowed Amanda to 'join in'.

Constructivist & sociocultural perspectives. From a constructivist viewpoint, co-operative and collaborative problem solving are seen to mediate cognition. Amanda, perhaps as a check on her own thinking (or as a way of being sociable), had looked at Gemma's written work. In answering Amanda, Gemma was alerted to reconsider or reflect on her thinking, and detected her mistake. There seemed mutual benefit in the two students' exchange.

From a sociocultural perspective, co-operation and collaboration are cast in slightly different terms. Social interaction is seen to directly constitute cognition (Cobb & Gravemeijer et al., 1997), so that being excluded from conversation with peers is highly detrimental. Whichever theoretical orientation is taken, we ask: did Amanda's early predicament of not having a working relationship with the students she was sitting next to, which was in part accounted for by her own actions, deserve notice by the teacher and call for intervention? Besides effects on cognition, if a student wants to participate, then lack of opportunity to do so could result in frustration and feelings of disempowerment (Tobin, 1998).

Trying Things Out on a Graphics Calculator and Explaining to the Class

Mr C:	Girls we might just stop there for a minute. . . . Okay that brings us to CD [part (c) of the question]. You could probably all do that? It wasn't all that difficult? What did you do?

Worked out the magnitude? So what's the answer? Naomi?

Naomi: 25

Mr C: So the magnitude is 25. And [going onto part (d) of the question] what is the direction? What is the direction of AB? How do you get direction? Who can help? What's the direction?

Jenny: What I did was go to my little Aplet that works out magnitude, and if you increase or decrease the i and j components in proportion, so "3 and -4" will be "6, and -8" and "9 and -12".

Mr C: What is Jenny doing?

Nicole: Trial and error.

Mr C: Yes, she is doing it trial and error but she is using a ratio, she is using the ratio of the two because that gives the same direction . . .

Jenny: . . . so I said 15 and -20 and it worked [resulting in the magnitude 25 for CD].

The "Aplet" or program was designed by Jenny and Katie on the first day that I observed the class, one month prior to this lesson. Its development and application has been the subject of another paper (Forster & Taylor, 1999a). The Aplet comprised a generalised equation ABS ([I, J]) = M, where ABS is the absolute value or magnitude function, so that to evaluate magnitude M only the vector components, I and J, needed to be entered (see Figure 2).

Figure 2. The symbolic and numeric screens of a graphics calculator for the ABS aplet.

Constructivist & sociocultural perspectives. Jenny made the connection that the vector components I and J and the vector magnitude M stay in proportion for vectors with a given direction. This proportional property is commonly encountered during early high-school years in Pythagorean triplets and with sine, cosine and tangent ratios. Jenny appeared to have *transferred* her knowledge of the property to the ABS function relationship. Here transfer is described in cognitive terms and as being between abstract tasks. Cobb and Bower's (1999) social constructivist view extends to accepting the sociocultural proposition that students *reason with technology*. Its use is social in the sense that students enter an interactive, intellectual partnership with it (Salomon & Perkins, 1998). From this perspective, transfer of knowledge involved Jenny recognising that her mathematising practices in one social context (an instructional setting without technology) were relevant to another social context (where interaction extended to using technology).

Also, Jenny's practice of experimenting with her graphics calculator can be cast, for her individually, as a transferable social practice of technology usage. "My little Aplet" perhaps indicates a feeling of ownership or empowerment by her, or "Technology as an extension of self ('Come fly with me!')" (Galbreith, Renshaw, Goos & Vincent, 1999, p. 225), with the tool being used to share and support reasoning in a spontaneous way. Whether between tasks or social contexts, the question is, how might transfer be encouraged? Familiarity with using the ABS function seemed relevant--Jenny and Katie used the ABS Aplet frequently from when they first designed it up until this episode nearly one month later. Katie followed the next week with another innovative approach (Forster & Taylor, 1999b).

Listening to the Teacher in Whole-class Work

After eliciting other methods from students for part (d) of the question (given at beginning of the analysis) the teacher continued:

Mr C: Right. So if I asked you, let's say if I asked for a vector that is parallel to CD, any vector that is parallel to CD [which was 7i - 24j], what would be your answer? Carla, what would you say?

Carla: 14i take 48j

Mr C: Correct. However, let's say I drew this one parallel down here, would this be the same vector?

Katie: Yes

The transcripts show that this episode was the only time that Carla participated verbally in whole-class discussion during the lesson. In my observation she was characteristically a non-contributor in the whole-class domain.

Constructivist & sociocultural perspectives. Sfard's (1998) acquisition metaphor (AM) for learning seems to encapsulate the practice of Carla learning predominantly by listening, in contrast to the participation metaphor (PM) which summarises the orientation of other students who regularly and voluntarily contributed to class discussion:

> While the AM stresses the individual mind and what goes 'into it' the PM shifts the focus to the evolving bonds between the individual and others. While AM emphasizes the inward movement of an object known as knowledge, PM gives prominence to the mutuality characteristic of the whole-part relation . . . The whole and the parts affect and inform each other. (pp. 6-7)

Constructivist, personal, reflective understanding can be seen to be consistent with Sfard's learning-as-acquisition, while reflexive enculturation associated with sociocultural views is congruent with learning-as-participation. Students' intuitive beliefs about the nature of learning might be located anywhere along the 'constructivist (individual) ↔ sociocultural (social)' continuum. Beliefs often are reflected in actions, are based on methods of instruction previously experienced, and can differ among students in a class (Tobin, Tippins & Hook, 1995).

Teachers' beliefs and actions also can determine the style of students' learning. We ask here, what does the evidence suggest as to the mode of learning encouraged in the Year 11 class? Quite frequently the teacher targeted answers, without asking or waiting for justification. For example, in the whole-class episode above-- "...what would be your answer?" and, from before, "What did you do? Worked out the magnitude? So what's the answer?" This practice has been recognized as problematic for engaging students in a discourse that reveals the depth of their understanding.

> If the teacher is thinking of a certain way to the right answer. . . If this pattern shows itself as a repeated pattern with the teacher, the students will come to a meta-learning, like: the teacher is always right and whatever we believe, it is the teacher who sets the agenda and decides what is right or wrong . . . the students will react on the basis of this knowledge, and consequently they will hold it futile to oppose. (Alro & Skovsmose, 1998, p. 46)

Summary

The above five brief classroom events, witnessed in the first 11 minutes of a lesson in the Year 11 class, illustrate diversity in students' involvement in learning mathematics through problem-solving. Using constructivism as a referent for analysis led us to ask: (1) Was input by more than one student needed in whole-class review work in order for the teacher to ascertain divergence in student understanding? (2) Was the teacher justified in moving on with his teaching agenda in view of a student's imprecise answer to a question and in view of the potentially problematic nature of students' tacit knowledge? and (3) Should an ambivalent response by a student to the teacher's suggestion not to use diagrams have been recognized as a signal that students might have benefited from their continued use?

Referencing sociocultural theory led us to ask: (1) Did the understanding that a single student constituted through interaction with the teacher in whole-class work necessarily extend to other students? (2) Did the teacher's omission to negotiate finer detail mask widespread disagreement about the concept of position vectors? (3) Were the set mathematical problem-solving tasks authentic? (4) Did a student's predicament of not having a working relationship with students she was sitting next to call for earlier teacher intervention? (5) How might teachers better encourage transfer? and (6) Which mode of learning was encouraged in view of the teacher's practice of targeting answers ?

Small-group Problem-solving

Now we consider another mode of problem-solving evidenced in the class: problem-solving in *small-groups*. We focus on the role of graphics calculators and the actions of one student, Christy. Selecting her was motivated by her responses subsequent to the problem-solving activity and her willing participation in the activity, which seemed to contrast markedly with her infrequent participation in whole-class work.

> After finishing 'working along with the class' on the problem described above, students moved on to other text-book exercises. The teacher and I distributed work that had been handed in for checking and discussed students' difficulties with them on a one-to-one basis.
>
> Mrs F: Now, where is your solution. [pause] The next step? Oh, you've got it out. Yes, that's the answer. So did you do it yourself?
>
> Christy: Well, we worked on it as a group. We did it together.

The requested solution was for a problem based on a practical task (see worksheet in Appendix 2) in which students used motion detectors to locate an 'invisible' object. A motion detector emits sonic waves which, when reflected from an object back to the detector, generate distance data in a data logger. A graphics calculator attached to the data-logger is used to control data collection and data are recorded on it. After setting up their solutions to the problem, students also used their calculators to process the data. The class worked in groups of four or five. The activity was introduced and discussed in terms of the motion detectors modeling medical equipment for locating brain tumors, and this application was described in a newspaper article attached to the handout. Students were asked to solve the problem in their groups and to hand in individual solutions.

The students organised themselves to take various roles, thereby experiencing the activity in differing ways. In a written response, Christy indicated that, from a choice of 'leader, helper, onlooker and distracter', she was a helper in the activity, and a helper and onlooker to solving the problem. Her responses match the evidence in the audio-recording. During the problem-solving, although she initially joined in discussing the diagram that one student was drawing for the group, she later became an onlooker, saying "I don't know how we are supposed to do this", but remained focussed on the activity. Then, she looked at how another group was solving the problem and came back to report that "They've got their detector perpendicular", but her group had not narrowed the task in this way. After some discussion, one of her group suggested using the cosine rule, and Christy responded: "But we haven't got an angle". With Christy watching, the others worked on the problem, and a short time later found an angle and finished the solution, just as the lesson was ending.

Constructivist view. The graphics calculator was used as a tool to generate data and then, after conceptualising the problem, students used the calculator for processing the data. Christy's willingness to be involved is indicated by her helping in the group activity and at the start of the problem solving. However, making successive links between the problem and her existing knowledge of the Pythagorean Theorem, trigonometric properties and then to finally

use vector components to locate the position of the 'tumor', seemed beyond her. Yet others managed the answer. So, can we say that 'listening' without participating when working problems along with the class, as was Christy's habit, is not effective in developing ability to think through problems, for all students?

Sociocultural view. A sociocultural view holds that cognition is contingent on participation: "[M]athematics 'exists' first on the interindividual plane, and only then is it internalised on the intra-individual plane" (Steffe, 1995, p. 494). But Christy did not participate in the rapid cut-and-thrust of whole-class debate so we ask what actions might a teacher take to enable students to develop the important linguistic resources for mathematical conversation, first at the level of the whole class and later in small groups? On the other hand, rather than looking to linguistic skills, the cause of students' non-participation might be due to uncertainty as to the class response and to the teacher not choosing them to answer. In the Year 11 class, students' confidence to participate in discussion in the whole-class forum seemed to attract as well as be a result of the teacher's invitational stance, a stance that wasn't directed toward all students (Forster & Taylor, 1999b).

In regard to using graphics calculators for processing data, entailing for example the Pythagorean and cosine rules that many had programmed into their calculators, students could be said to have used the technology as one would a servant: "('Tell the thing what to do!')" (Galbraith, Renshaw, Goos & Geiger, 1999). But Christy did not share the role of producing or checking the group answers, which raises the question of whose responsibility is it that group members all contribute? Anderson, Holland and Palincsar (1997) suggest that to optimise the outcomes from group work, students should be made accountable "for both group learning and performance, for both group process and final product, and for both individual and collective success" (p. 379).

Concluding Discussion

In this paper, we have inquired into a segment of teaching and learning associated with a teacher working a problem along with the class, and have considered briefly the alternative of small-group problem-solving. The choice of how to facilitate problem-solving is not trivial. It has been identified by the U.S. Department of Education (1999) as a factor in the high performance by Japanese students relative to U.S. and German students in the Third International Mathematics and Science Study (TIMSS). Based on a video-tape study of 100 German, 50 Japanese and 81 US eighth-grade classrooms it was concluded that teaching methods vary dramatically between East and West:

> U.S. and German lessons tend to have two phases, the teacher demonstrates and/or explains how to solve an example problem . . . the goal in both countries is to teach students a method for solving the example problem(s). In the application phase, students practice solving examples on their own while the teacher helps individual students who are experiencing difficulty. . . In Japanese lessons the order of activity is generally reversed. Problem solving comes first, followed by a time in which students reflect on the problem, share the solution methods they have generated, and jointly work to develop explicit understandings of the underlying mathematics concepts. . . . Japanese teachers, in certain respects, come closer to implementing the spirit of current ideas advanced by U.S. reformers than do U.S. teachers. (U.S. Department of Education, 1999, p. 5-7)

In relation to current reform which largely seems guided by *constructivist* principles (e.g., Curriculum Council of Western Australia, 1998; NCTM, 1989, 1991, 1995), the teaching method identified in this study, 'working a problem along with the class', could be described ostensibly as less didactic and more participatory than the 'worked *example*' approach that is portrayed as characteristic of U.S. and German classrooms. On the other hand, our analysis

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suggests that 'working a problem along with the class' can place less emphasis on both individual student thinking and on the social aspect of students explaining their thinking than is depicted in the study for Japanese mathematics classes.

From our examination of teaching practices in a Year 11 class, where we have emphasised both cognitive and social knowledge construction, we recommend that students' problem-solving abilities might be improved if they engage in *reflective thinking* and *rich conversation*. We suggest that the following teaching actions in the whole-class forum might help to create the right conditions for this type of learning: (1) The teacher requiring full explanations rather than assuming that students understand and are in agreement; (2) many rather than few students being involved in class discussion; (3) careful listening by the teacher when student explanations or suggestions are offered; (4) the teacher being mindful of students sitting next to someone with whom they can collaborate and (5) avoiding questions that target only narrowly-defined answers. Although small-group work can provide opportunities for more students to speak, our cursory analysis illustrated the possibility of minimal participation. However, even for students on the periphery, small-group learning might involve greater active involvement than whole-class work, provided students have developed adequate linguistic resources to engage in conversation.

In Japan there is an emphasis on students deriving alternative problem-solving methods. Alternatives considered in this paper included diagrammatic approaches, on which some students might rely in preference to algebraic approaches, and the utilisation of technology. Graphics calculators have been cast as the potentially empowering 'extension of self', as a 'servant' for calculation and as a tool that records measurements. In this inquiry into learning, through purposely selecting data that reflected graphics calculator usage, we have construed graphics calculators as a potentially valuable learning resource, but we note the comment in the video-study report that: "we never observed calculators being used in a Japanese classroom" (U.S. Department of Education, 1999, p. 7).

Our dual theoretical approach has alternately focussed on cognitive then social aspects of teaching and learning. The underlying assumptions of constructivism and sociocultural theory are sometimes inconsistent, but we have found both perspectives valuable in allowing complementary insights into the apparently problematic nature of teaching and learning in a Year 11 mathematics class. To finish, we make a recommendation that was discussed with the teacher. Students' approaches to learning, including their active participation in class, are interactively determined by themselves, the teacher and other class members. Critical awareness is needed by teachers of how their own actions mediate the participation in class of each of their students.

References

Alro, H., & Skovsmose, O. (1998). That was not the intention! Communication in mathematics education [1] *For the learning of mathematics, 18*(2), 42-51.

Anderson, C. W., Holland, J. D. & Palincsar, A. S. (1997). Canonical and sociocultural approaches to research and reform in science education: The story of Juan and his group. *The Elementary School Journal, 97*(4), 359-383.

Anderson, J. R., Reder, L. M., & Simon, H. A. (1997). Situated versus cognitive perspectives: Form versus substance. *Educational Researcher, 25*(4), 5-11.

Atkinson, P., & Hammersley, M. (1994). Ethnography and participant observation. . In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of qualitative research* (pp. 248-261). Thousand Oaks, CA: SAGE.

Cobb, P. (1994). Where is the mind? Constructivist and sociocultural perspectives on mathematical development. *Educational Researcher, 23*(7), 13-20.

Cobb, P., & Bowers, J. (1999). Cognitive and situated learning perspectives in theory and practice. *Educational Researcher, 28*(2), 4-15.

Cobb, P., Boufi, A., McClain, K., & Whitenack, J. (1997). Reflective discourse and collective reflection. *Journal of research into mathematics education, 28*(3), 258-277.

Cobb, P., Gravemeijer, K., Yackel, E., McClain, K., & Whitenack, J. (1997). Mathematizing and symboCarlaing: The emergence of chains of significance in one first-grade classroom. In D. Kirshner & J. A. Whitson (Eds.), *Situated cognition: Social, semiotic, and psychological perspectives* (pp. 151-234). Mahwah, NJ: Lawrence Erlbaum.

Cohen, L., & Manion, L. (1994). *Research methods in education* (4th ed.). London: Routlege.

Curriculum Council (1998). Curriculum framework for Kindergarten to Year 12 education in Western Australia. Perth, Western Australia: Curriculum Council

Davis, B. (1996). *Teaching mathematics: Toward a sound alternative*. New York, NY: Garland

Erickson, F. (1986). Qualitative methods in research on teaching. In M. C. Wittrock (Ed.), *The handbook of research on teaching* (3rd ed., pp. 119-161). New York: Macmillan.

Ernest, P. (1995). The one and the many. In L. P. Steffe & J. Gale (Eds.), *Constructivism in education* (pp. 459-486). Hillsdale, NJ: Lawrence Erlbaum.

Eylon, B. S. & Linn, M. C. (1988). Learning and instruction: An examination of four research perspectives in science education. *Review of Educational Research, 58*, 251-301.

Fenstermacher, G. D. (1994). The knower and the known: The nature of knowledge in research on teaching. In L. Darling-Hammond (Ed.), *Review of research in education* (pp. 3-56). Washington, DC: AERA Forster.

Forster, P. & Taylor, P., (1999a). *Learning mathematics through conversation and utilising technology*. Paper presented at the annual conference of the Australian Association for Educational Research (AARE), Melbourne, Vic. http://www.swin.edu.au/aare/99pap/for99012.htm

Forster P. A., & Taylor P. C. (1999b). *A multiple perspective analysis of learning in the presence of technology*. Manuscript submitted for publication.

Galbreith, P., Renshaw, p., Goos, M., & Geiger, V. (1999). Technology, mathematics, and people: Interactions in a community of practice. In J. M. Truran & K. M. Truran (Eds.), *Making the difference: Proceedings of the twenty-second annual conference of the Mathematics Education Group of Australasia* (pp. 223-230). Adelaide.

Gergen, K. J. (1995). Social construction and educational process. In L. P. Steffe & J. Gale (Eds.), *Constructivism in education* (pp. 17-40). Hillsdale, NJ: Lawrence Erlbaum.

Greeno, J. G. (1997). On claims that answer the wrong questions. *Educational Researcher, 26*(1), 5-17.

Guba, E. G., & Lincoln, Y. S. (1989). *Fourth generation evaluation*. Newbury Park, CA: SAGE.

Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of research in mathematics teaching and learning* (pp. 65-97). New York: Macmillan.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge: The Press Syndicate of the University of Cambridge

Livingston E. (1987). *Making sense of ethnomethodology*. London: Routledge.

National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics (1991). *Professional standards for teaching mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics (1995). *Assessment standards for school mathematics*. Reston, VA: Author.

Noddings, N. (1990). Constructivism in mathematics education. In R. B. Davis, C. A. Mayer, & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics* (pp. 7-29). Reston, VA: National Council of Teachers of Mathematics.

Richardson, L. (1998). Writing a method of inquiry. In N. K. Denzin & Y. S. Lincoln (Eds.), Collecting and interpreting qualitative materials (pp. 345-371). Thousand Oaks, CA: Sage.

Roth, W. -M., & Tobin, K. (in press). College physics teaching: From boundary work to border crossing and community building. In P. C. Taylor, P. J. Gilmer, & K. G. Tobin (Eds.), *Transforming Undergraduate Science Teaching: Constructivist Perspectives*. Philadelphia, PA: Lang Publishers.

Sadler, A. J. (1993). *Geometry and Trigonometry*. Perth, WA: Sadler Family Trust.

Salomon, G., & Perkins, D. N. (1998). Individual and social aspects of learning. In P. D. Pearson, & A. Iran-Nejad (Eds.), *Review of research in education* (Vol. 23, pp. 1-70). Washington, DC: American Educational Research Association.

Schön, D. A. (1987). *Educating the reflective practitioner*. San Francisco, CA: Jossey-Bass.

Sfard, A. (1998). On two metaphors for learning and the dangers of choosing just one. *Educational Researcher, 27*(2), 4-13.

Sfard, A., Nesher, P., Streefland, L., Cobb, P., & Mason, J. (1998). Learning mathematics through conversation: Is it as good as they say? [1]. *For the Learning of Mathematics, 18*(1), 41-51.

Steffe, L. P. (1995). Alternative epistemologies: An educator's perspective. In L. P. Steffe & J. Gale (Eds.),

BEST COPY AVAILABLE

Constructivism in education (pp. 489-523). Hillsdale, NJ: Lawrence Erlbaum.
Taylor, P. C. (1996). Mythmaking and mythbreaking in the mathematics classroom. *Educational Studies in Mathematics 31*(1-2), 151-173.
Taylor, P. C., Fraser, B. J., & Fisher, D. L. (1997). Monitoring constructivist classroom learning environments. *International Journal of Educational Research, 27*(4), 293-302.
Tobin, K. (1998). Issues and trends in the teaching of science. In B. J. Fraser & K. G. Tobin (Eds.), *International handbook of science education* (pp. 129-151). Dordretch, The Netherlands: Kluwer Academic.
Tobin, K., Tippins, D. J., & Hook, K. S. (1995). Students' Beliefs About Epistemology, Science and Classroom Learning: A Question of Fit. In S. M. G. Linn & R. Duit (Eds.), *Learning Science in the Schools: Research Reforming the Practice* (pp. 85-108).
U.S. Department of Education (1999). *The TIMSS videotape classroom study: Methods and findings from an exploratory research project on eighth grade mathematics instruction in Germany, Japan, and the United States*. Washington, DC: U.S. Government. Available: http://nces.ed.gov/timss
van Manen, M. (1990). *Researching lived experience, Human science for an action sensitive pedagogy*. London: State University of New York Press.
Voigt, J. (1994). Negotiation of mathematical meaning and learning mathematics. *Educational Studies in Mathematics, 26*, 275-298.
Von Glasersfeld, E. (1987). Learning as a constructive activity. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 3-18). Hillsdale, NJ: Lawrence Erlbaum Associates.

Appendix 1: Questionnaire
In this Maths class I actually learn by:

Students were asked to select from a scale, 'Often', 'Sometimes', 'Seldom', 'Never' in answer to the following questions, and then to 'Please put a tick next to the four ways that you most prefer to learn by.'

I actually learn by:
1. Answering questions in whole class work.
2. Explaining things to the class.
3. Answering the teacher's questions, one-to-one.
4. Explaining my work to the teacher, one-to-one.
5. Answering friends' questions.
6. Explaining work to friends.

I actually learn by:
7. Asking the teacher if he agrees with my ideas in whole-class work.
8. Asking for an explanation in whole-class work.
9. Asking the teacher, one-to-one, if he agrees with my ideas.
10. Asking the teacher, one-to-one, to explain things.
11. Asking friends if they agree with my ideas.
12. Asking friends to explain things.

I actually learn by:
13. Listening to the teacher in whole-class work.
14. Listening to the teacher explain things to me one-to-one.
15. Listening to my friends explain things.
16. Working on problems along with the class.
17. Working with other students on problems.
18. Working by myself.

19. I like using my graphics calculator.
20. My graphics calculator is easy to use.
21. Solving problems on a graphics calculator is doing proper mathematics.
22. Using a graphics calculator helps me understand mathematics.
23. A graphics calculator helps me explain mathematics.

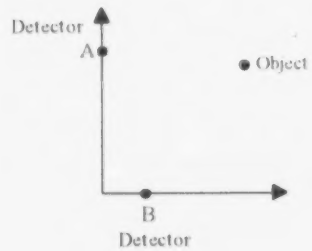
Appendix 2: Worksheet

Locating an Object in No-(Wo)man's Land

Graphics Calculator Activity

X-raying a tooth to determine if a filling is necessary, satellite spying on the enemy, and prospecting for underground minerals are some of the many instances where technologies use reflected radio or sound waves to get images of objects that are not immediately accessible. The following activity explains how these work.

1. Set up axes in a cleared space and place an object in the quadrant, as shown, not close to the axes.



2. Place the detector at A, anywhere on the *y* axis, and measure the distance to A from the origin. MONITOR the position of the object by turning the detector until a stable reading is obtained. Record the distance.

3. Repeat with the detector at a point B on the *x* axis.

4. Calculate the co-ordinates of the position of the object.

While motion detectors use sonic (sound) waves for measuring distance, the *Stealth Station Treatment Guidance Platform* (see accompanying article), used for detecting tumors, uses electromagnetic waves travelling at 299 000 kilometers per second. Wave pulses emitted from detectors are reflected off a tumor, back to the detector. The time between emission and receiving the reflected pulse back is used to calculate distance.

5. Imagine that the diagram above represents the examination by a neurosurgeon of a tumor located at the point marked 'object'. The detectors are placed quite close to the person's skin at A (0, 3) and B (1, 0), where measurements are in cm. For the detector at A, the time between emission and receiving back a wave pulse is 1.6019×10^{-10} seconds. For the detector at B, the time is B is 2.0940×10^{-10} seconds. Locate the position of the tumor.

In reality, most detection situations involve three-dimensional space and you need three detectors to locate position and a fourth one as a check for error. Using the GPS (Global Positioning System) requires signals from four satellites to reach a receiver in order to accurately locate position, anywhere on Earth. There are 24 satellites around the Earth that make up the GPS system.